A.3 Matrix Inverses

This is a guide to inverting 1×1, 2×2, and $n \times n$ matrices.

Let A be the 1×1 matrix

$$A = \begin{bmatrix} a \end{bmatrix}. \tag{A.44}$$

The inverse is simply the reciprocal:

$$A^{-1} = \begin{bmatrix} 1/a \end{bmatrix}. \tag{A.45}$$

Let B be the 2×2 matrix

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}. \tag{A.46}$$

It can be shown that the inverse follows a simple pattern:

$$B^{-1} = \frac{1}{\det B} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix} \tag{A.47}$$

$$= \frac{1}{b_{11}b_{22} - b_{12}b_{21}} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}. \tag{A.48}$$

Let C be an $n \times n$ matrix. It can be shown that its inverse is

$$C^{-1} = \frac{1}{\det C} \operatorname{adj} C, \tag{A.49}$$

where adj is the **adjoint** of C.

A.4 Euler's Formulas

Euler's formula is our bridge back-and forth between trigonomentric forms ($\cos\theta$ and $\sin\theta$) and complex exponential form ($e^{j\theta}$):

$$e^{j\theta} = \cos\theta + j\sin\theta. \tag{A.50}$$

Here are a few useful identities implied by Euler's formula.

$$e^{-j\theta} = \cos\theta - j\sin\theta \tag{A.51a}$$

$$\cos\theta = \Re(e^{j\theta}) \tag{A.51b}$$

$$= \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \tag{A.51c}$$

$$\sin\theta = \Im(e^{j\theta}) \tag{A.51d}$$

$$= \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right). \tag{A.51e}$$

A.5 Laplace Transforms

B5

The definition of the one-side Laplace and inverse Laplace transforms follow.

Definition A.1

Laplace transforms (one-sided)def:laplace-transforms Laplace transform $\mathcal{L}$:

$$\mathcal{L}(y(t)) = Y(s) = \int_0^\infty y(t)e^{-st}\,dt. \tag{A.52}$$

Inverse Laplace transform $\mathcal{L}^{-1}$:

$$\mathcal{L}^{-1}(Y(s)) = y(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} Y(s)e^{st}\,ds. \tag{A.53}$$

See **??** for a list of properties and common transforms.