4.10 Problems

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Problem 4.1 TRIANGLE Respond to the following questions and imperatives with one or two sentences and, if needed, equations and/or sketch.

1. Why do we include a resistor in lumped-parameter motor models?
2. How are brushes used in brushed DC motors?
3. With regard to standard motor curves, why do we say the "braking power" is equivalent to the power that could be successfully transferred by the motor to the mechanical system?
4. In terms of electrical and mechanical processes, why does an *efficiency* versus torque motor curve have a peak?
5. As a DC motor's bearings wear down, how will its efficiency curve be affected?

Problem 4.2 SQUARE Consider the system presented in the schematic of **??**. Let the DC motor have motor constant K_a (units N-m/A) and let the motor be driven by an ideal *current* source I_S. Assume the motor inertia has been lumped into J_1 and motor damping lumped into B_1.

1. Draw a linear graph model.
2. Draw a normal tree.
3. Identify any *dependent* energy storage elements. If the motor was driven by an ideal voltage source instead, how would this change?

Draw a linear graph model and normal tree.

Problem 4.3 RECTANGLE Consider the system presented in the schematic of **??**. From the linear graph model and normal tree derived in problem 4.2, derive a state-space model in standard form. Let the outputs be θ_{J_1} and θ_{J_2}, the angular positions of the flywheels.

Problem 4.4 QUADRILATERAL Consider the linear graph model of a motor coupled to a rotational mechanical system shown in **??**. This is similar to the model from

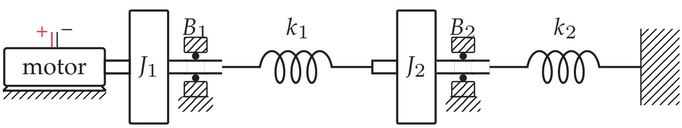


Figure 4.18. schematic of an electromechanical system for .

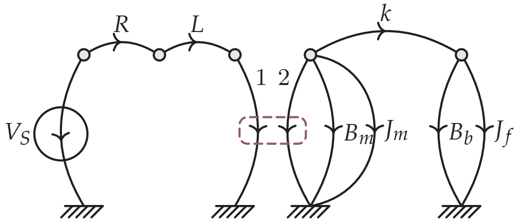


Figure 4.19. a linear graph model of the electromechanical system.

the **??**, but includes the flexibility of the shaft coupler. An ideal voltage source drives the motor—modeled as an ideal transducer with armature resistance R and inductance L, given by the manufacturer in table 4.1. The ideal transducer's rotational mechanical side (2) is connected to a moment of inertia J_m modeling the rotor inertia and damping B_b modeling the internal motor damping, both values given in the motor specifications. Take $B_b = B_m$ and $J_f = 0.324 \cdot 10^{-3}$ kg-m^2. Assume the shaft coupling has a torsional stiffness of $k = 100$ N-m/rad.

1. Derive a state-space model for the system with outputs i_1 and Ω_{J_f}.
2. Create a Matlab `ss` model for the system and simulate its response from rest to an input voltage $V_S = 10$ V.
3. Plot the outputs through time until they reach steady state.

Problem 4.5 MRPOTATOHEAD Consider the linear graph model (with normal tree) of **??**. This is a model of a motor with constant K_a connected to a pair of meshing gears with transformer ratio N, the output over input gear ratio. An ideal voltage source drives the motor—modeled as an ideal transducer with armature resistance R and inductance L. The motor's rotational mechanical side (2) is connected to a moment of inertia J_2 modeling the rotor and drive gear combined inertia. The damping element B_2 models the internal motor damping and the drive gear bearing damping. The output side of the gear transducer (4) is connected to a moment of inertia J_4 modeling the output gear and load combined inertia. The damping element B_4 models the internal motor damping and the drive gear bearing damping. Use the parameter values given in problem 4.5.

1. Derive a state-space model for the system with outputs Ω_{J_2} and Ω_{J_4}.
2. Create a Matlab `ss` model for the system and simulate its response from rest to an input voltage $V_S = 20$ V.
3. Plot the outputs through time until they reach steady state.

R	2 Ω
L	8 mH
K_a	0.2 N-m/A

J_2	$0.1 \cdot 10^{-3}$ kg-m^2
B_2	50 μN-m/(rad/s)
N	5
J_4	$1 \cdot 10^{-3}$ kg-m^2
B_4	70 μN-m/(rad/s)

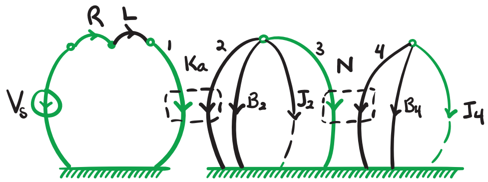


Figure 4.20. A linear graph model with normal tree in green of an electromechanical system with a gear reduction.

Problem 4.6 CLUNKER Draw a *linear graph*, a *normal tree*, identify *state variables*, identify *system order*, and denote any *dependent energy storage elements* for each of the following schematics.

1. The electronic system of figure 4.21, voltage and current sources, and transformer with transformer ratio N.
2. The electromechanical system of figure 4.22 with motor model parameters shown, coordinate arrow in green. Model the propeller as a moment of inertia J_2 and damping B_2.
3. The translational mechanical system of figure 4.23, force source, coordinate arrow in green.

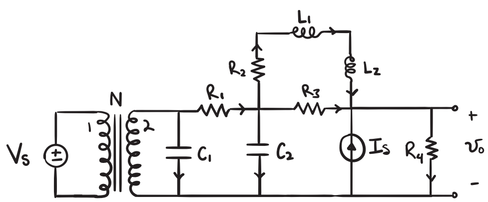


Figure 4.21. a circuit diagram.

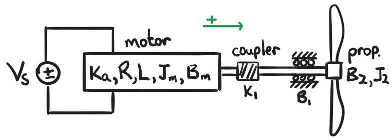


Figure 4.22. Sketch of a motor coupled to a fan.

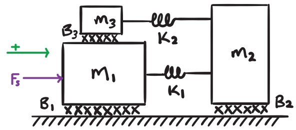


Figure 4.23. Schematic of a mechanical system.

Problem 4.7 CURVY Consider the DC motor curves of figure 4.13, reproduced in figure 4.24.

1. At peak efficiency, what is the steady-state motor speed?
2. At peak efficiency, what is the steady-state motor torque?
3. You are to use this motor to drive a load at a constant angular speed of 100 rad/s with at least 1 N-m of torque. You wisely choose to use a gear reduction between the motor and load. What should the gear ratio be to meet the above requirements and optimize efficiency? Justify your answer in terms of the motor curves of figure 4.24.

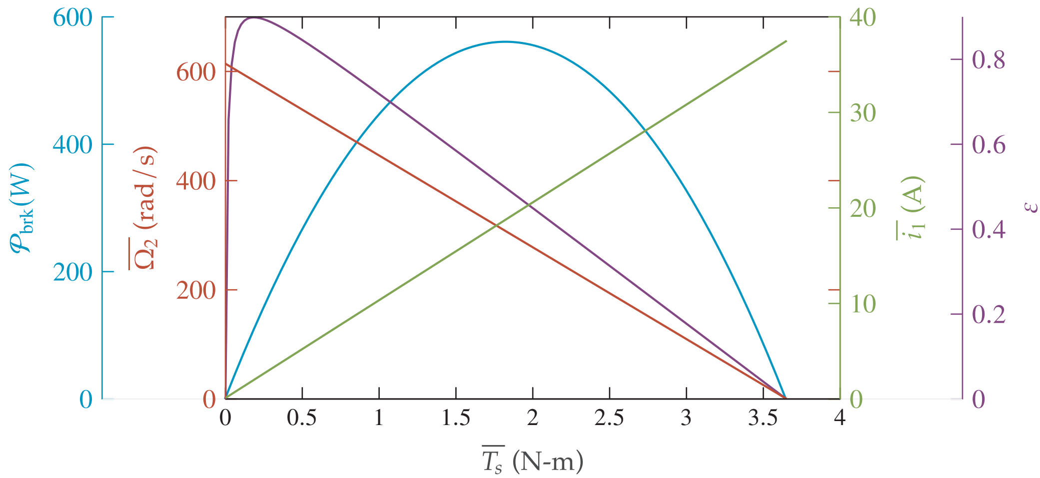


Figure 4.24. The motor curve figure 4.13.

Problem 4.8 CHAIR Consider the opamp circuit of figure 4.25, which will be used to drive a PMDC motor. The input can supply a variable $V_S \in [0, 10]$ V, the motor has constant $K_a = 0.05$ V/(rad/s) and coil resistance $R_m = 1\ \Omega$, and the opamp has differential supplies ±24 V. Assume the maximum torque magnitude required from the motor at top speed is $|T_2| = 0.1$ N-m and ignore any voltage drop in the motor due to the coil inductance.[13] Select R_1 and R_2 to demonstrably meet the following *design requirements*:

1. drivable motor speeds of at least $[0, 400]$ rad/s,
2. no saturation of the opamp (i.e. $|v_o| < 24$ V), and
3. a maximum combined power dissipation by R_1 and R_2 less than 300 mW.

13. Do not ignore the voltage drop across R_m, though. Note that this amounts to an assumption of steady-state operation at top speed. By requiring a specific T_2, we are also implicitly ignoring torque losses due to motor bearing damping.

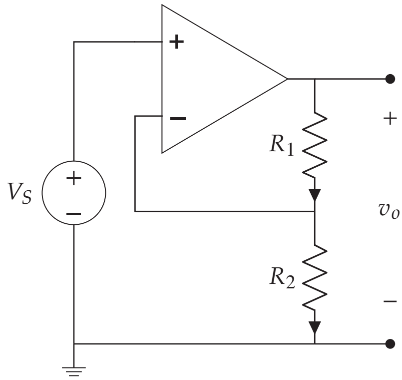


Figure 4.25. An opamp circuit.

Hint: start with the elemental equations of the DC motor to determine the necessary amplifier output v_o, then constrain R_1 and R_2 to meet the gain requirements, and finally further constrain R_1 and R_2 to meet the power dissipation requirement.

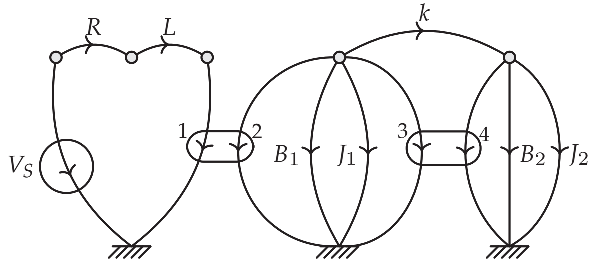
Problem 4.9 ONOMATOPOEIA Consider the DC motor curves of figure 4.13, reproduced in figure 4.24. If this motor is running at 400 $\frac{\text{rad}}{\text{s}}$,

1. How much torque is produced?
2. What is the output power?
3. What is the input power?
4. Why are the input and output power the same or different?

Problem 4.10 DEGLAZIFICATION Explain in your own words what lumped parameter elements should be used when modeling an electric motor and why.

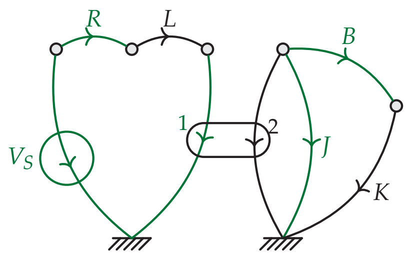
Problem 4.11 CONFUZZLED In the linear graph below a system is depicted consisting of a motor with its related damping and inertia driven by a voltage source and connected to a set of gears driving a second inertia. A rotary spring is attached between the two inertias.
Given this linear graph:

1. draw a normal tree,
2. determine the state variables and system order, and
3. list any dependent energy storage elements and explain what this implies.



Problem 4.12 LEVITATION In the linear graph and normal tree below a system is depicted consisting of a motor driven by a voltage source V_S with inertia J driving a rotary damper and spring connected in series. Let the motor constant be Ka, and outputs of the system be the rotational velocity of the inertia, Ω_J, and the change in rotational velocity across the rotational damper, Ω_B.
Given this linear graph and normal tree:

1. determine the state variables,
2. define the state, input, and output vectors,
3. write the elemental, continuity, and compatibility equations, and
4. solve for the state and output equations.



5 Linear Time-Invariant System Properties

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In this chapter, we will extend our understanding of linear, time-invariant (LTI) system properties. We must keep in mind a few important definitions.

The **transient response** of a system is its response during the initial time-interval during which the initial conditions dominate. The **steady-state response** of a system is its remaining response, which occurs after the transient response. Figure 5.1 illustrates these definitions.

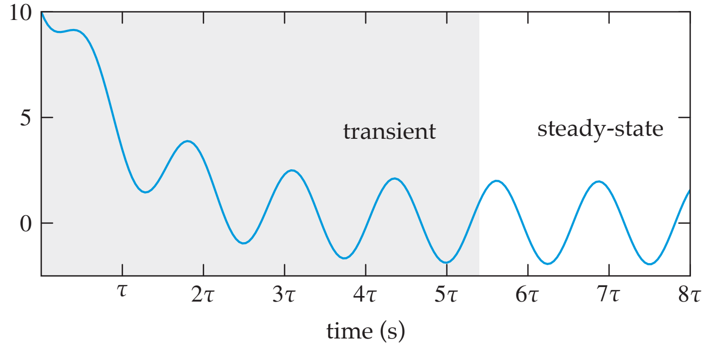

Figure 5.1. Transient and steady-state responses. Note that the transition is not precisely defined.

The **free response** of a system is the response of the system to initial conditions—*without forcing* (i.e. the specific solution of the io ODE with the forcing function identically zero). This is closely related to, but distinct from, the transient response, which is the free response *plus* an additional term. This additional term is the **forced response**: the response of the system to a forcing function—*without initial conditions* (i.e. the specific solution of the io ODE with the initial conditions identically zero).

5.1 Superposition, Derivative, and Integral Properties

From the principle of **superposition**, **linear, time invariant** (LTI) system responses to both initial conditions and nonzero forcing can be obtained by summing the free response y_{fr} and forced response y_{fo}:

$$y(t) = y_{\text{fr}}(t) + y_{\text{fo}}(t).$$

Moreover, superposition says that any linear combination of inputs yields a corresponding linear combination of outputs. That is, we can find the response of a system to each input, separately, then linearly combine (scale and sum) the results according to the original linear combination. That is, for inputs u_1 and u_2 and constants $a_1, a_2 \in \mathbb{R}$, a forcing function

$$f(t) = a_1 u_1(t) + a_2 u_2(t)$$

would yield output

$$y(t) = a_1 y_1(t) + a_2 y_2(t)$$

where y_1 and y_2 are the outputs for inputs u_1 and u_2, respectively.

This powerful principle allows us to construct solutions to complex forcing functions by decomposing the problem. It also allows us to make extensive use of existing solutions to common inputs.

There are two more LTI system properties worth noting here. Let a system have input u_1 and corresponding output y_1. If the system is then given input $u_2(t) = \dot{u}_1(t)$, the corresponding output is

$$y_2(t) = \dot{y}_1(t).$$

Similarly, if the same system is then given input $u_3(t) = \int_0^t u_1(\tau) d\tau$, the corresponding output is

$$y_3(t) = \int_0^t y_1(\tau) d\tau.$$

These are sometimes called the **derivative** and **integral properties** of LTI systems.